

THREE DIMENSIONAL ACES MODELS FOR BRIDGES

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SYNOPSIS

Plane grillage models are widely used for the design of bridges, and are suitable where bridges can be characterised as having a beam and slab arrangement, but have limitations when box girder sections are required. For box girders, a three dimensional analysis is necessary, which usually involves using finite elements. There are many practical difficulties using finite elements as a direct design tool, such as the size of model required to get sufficient accuracy, the difficulty of converting what are generally stress outcomes to design action parameters, and the limitations of computers, software and/or available design time.

This paper shows how a three dimensional arrangement of members, similar to those which would be used in a plane grillage, can provide direct design actions such as bending moments and shear forces, for any bridge type from a simple beam and slab through to a haunched continuous box girder. The method is as accurate as a fine mesh finite element model, but requires considerably less computation power, which means even large structures can be modelled with a relatively small program like ACES. A box girder example is worked through to illustrate how the analysis method may be applied and highlight the advantages of this method.

In conjunction with the three dimensional model, a new simple approach for treating shear lag is suggested. Both the shear lag analysis, and the three dimensional grillage method are very “transparent” about the interactions of the various parts of the structure carrying the loads, and both should also prove useful as teaching aid.

1. INTRODUCTION

The box girder is an efficient form of construction for bridges because it minimises weight, while maximising flexural stiffness and capacity. Single cell box girders, multi-cell box girders, and multiple box girders are all common forms of bridge construction.

The behaviour of box girders under eccentric loading is complex. Although the box shape is torsionally stiff, it tends to undergo distortion. Three dimensional analysis is generally required in order to accurately model the distortional behaviour. Finite element analysis can be used, however the models are time consuming to set up, difficult to modify and give stress outputs which need to be converted to moments and shears for design purposes.

This paper briefly describes a new three dimensional frame method, based on the principles of plane grillage analysis, which can be used to accurately model the complex behaviour of box girders, while retaining the simplicity and usefulness associated with plane grillages. A new

method for quantifying the effects of shear lag is suggested, which may be used to refine models constructed using the new three dimensional frame method.

2. BEHAVIOUR OF BOX GIRDERS UNDER ECCENTRIC LOADING

This section provides a brief overview of the behaviour of box girders under load. Much of the information in this section is covered in more detail by Hambly(1).

An eccentric load on a box girder may be separated into a symmetric component and an antisymmetric component. The symmetric component causes pure flexural and shear deformation which may be calculated using simple beam theory. The antisymmetric component has two effects on the girder, namely torsion and distortion. Torsion is a result of shear flow around the box, whereas distortion is the result of shear flow towards opposing corners of the box causing in-plane and out-of-plane flexure of the webs and flanges. Figure 1 shows how flexure, shear, torsion and distortion arise from the eccentric loading of a single cell box.

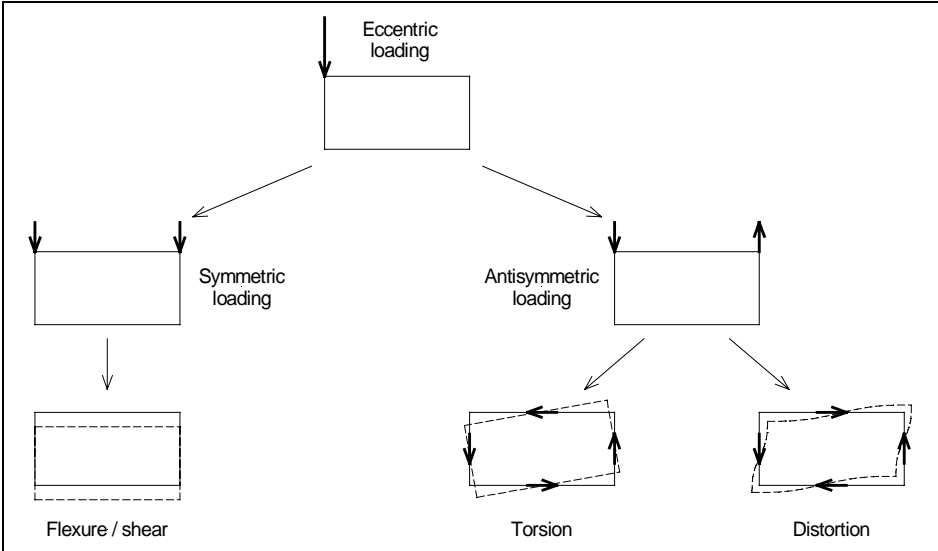


Figure 1: Behaviour of a single cell box girder under eccentric loading

The true rotation stiffness (C_{dt}) of a box girder is the combination of pure torsion stiffness (C_t) and distortion stiffness (C_d) as given by Equation (1).

$$\frac{1}{C_{dt}} = \frac{1}{C_d} + \frac{1}{C_t} \quad (1)$$

The pure torsion stiffness of a box section is readily quantifiable and can be calculated using equation (2).

$$C_t = \frac{4A_e^2}{\oint \frac{ds}{t}} \quad (2)$$

where:

A_e = the area enclosed by the box
s = the perimeter of the area enclosed by the box
t = the thickness of the plates comprising the box.

Box girders are generally stiff in pure torsion, however they can be flexible for distortion. Unlike torsion, distortion behaviour of box girders is more complex and considerably more difficult to predict accurately. It is a function of the in-plane and out-of-plane flexural stiffness of the webs and flanges comprising the box, the box dimensions, and the restraint of the box.

It should be noted that although it is possible to increase the true rotational stiffness of box girders and improve transverse load distribution by adding cross bracing and diaphragms, extremely torsionally and distortionally stiff sections may not distribute load well between bearings and may be sensitive to differential settlement of adjacent bearings.

3. ANALYTICAL METHODS AND BOX GIRDERS

3.1 Mathematical Methods

Mathematical methods have been developed for assessing the distortion behaviour of simple box girders, such as the beams-on-elastic-foundations method, however these methods can be complex for anything other than simple structures.

3.2 Plane Grillage Analysis

Plane (or two dimensional) grillage analysis is commonly used within the industry to assess the global effects of bridge decks, because it provides a relatively simple method for generating direct design actions for complex structures such as multiple box, multiple span decks. Analysis of box girder structures using plane grillages is limited, because the three dimensional behaviour is difficult approximate in two dimensions.

For single cell box or multiple box girders, it is possible to model each box as a single beam with bending, shear and torsion stiffness. The difficulty arises in assessing the true torsion (or rotation) stiffness of the box girder, which is comprised of pure torsion and distortion, as previously described. Equation (2) may be used to calculate the pure torsion stiffness, but what about distortion?

For a well braced box girder, it may be accurate enough to assume that the true rotation stiffness is approximately equal to the pure torsion stiffness, but for unbraced girders comprised of thin plates the true value is likely to be only a fraction of the pure torsion value. Mathematical methods can be used to estimate the true value, or it may be adequate to adopt a percentage of the pure torsion value as the true stiffness. A sensitivity analysis could be carried out for rotation stiffness values ranging from zero to the pure torsion value. These approaches are approximate at best and do not allow the designer to see how the structure is actually behaving.

It is also possible to model box girders by subdividing the box into individual beams corresponding to each web, however there is still the problem of assessing the distortional behaviour.

3.3 Space Frame Analysis

Space frame models can be used to model the distortion behaviour of box girders and although three dimensional models can be more time consuming to set up than two dimensional grillages, in some instances it can make the interpretation of results easier.

Current space frame modelling techniques include the Truss Space Frame, McHenry Lattice and Cruciform Space Frame (refer to Hambly(1) for more information about these methods).

Cruciform models can be quite accurate, even when relatively coarse divisions are used, however some interpretation of the results to obtain design actions is required.

3.4 Three Dimensional Finite Element Analysis

Finite element analysis has the ability to quite accurately model box girder structures, however it has the following limitations:

- Considerable design time must be spent to set up, modify and run the model;
- A fine mesh is required for accuracy, therefore model sizes are often very large;
- Outputs are often in the form of stresses, which generally have to be converted to design actions such as moments and shear forces for design purposes; and
- Current software packages are expensive and require powerful computers.

As a result, FEA is generally limited to assessing local effects and not complete structures.

4. NEW THREE DIMENSIONAL FRAME ANALYSIS METHOD

4.1 The Concept

A new three dimensional frame analysis method is proposed to accurately model the complex behaviour of box girders, while still achieving the simplicity and usefulness associated with two dimensional grillage methods. The idea is based on converting a plane grillage to three dimensions.

As is the case for a plane grillage, the structure is divided into main longitudinal beams, with an appropriate number of beams selected to model the total vertical stiffness of the section. In the case of a symmetrical single cell box girder, two beams are selected coincident with each web, each having a shear area (A_y) equal to the area of the web and a bending stiffness about the horizontal (y) axis (I_y) equal to half the total bending stiffness of the section.

Similarly, a number of longitudinal beams are selected to model the total horizontal stiffness of the section. For a single cell or multi-cell box girder, generally two beams will be sufficient, one each to represent the top and bottom flanges. The top beam has a shear area equal to the area of the top flange, the bottom beam has a shear area equal to the area of the bottom flange, and the sum of the bending stiffness about the vertical (z) axis (I_z) of the two beams is equal to the total bending stiffness of the section.

It is critical that longitudinal beams are modelled to have no compression area. This ensures the flexural behaviour of the structure is modelled by bending of beam members alone and does not include eccentric axial stiffness effects.

The main longitudinal beams are connected by frame members representing the individual stiffness of the flange and web elements in the direction orthogonal to the main beams. For bending in the same plane as the main beam flexure, the frames are “infinitely stiff” (rigid), which projects the main flexural beam curvature to the correct strain at the corners of the box. The longitudinal spacing of frames is not critical, but more frames improves the accuracy of the model and an even spacing of frames will reduce the number of member property types that need to be defined.

Longitudinal distribution by the slab members (flanges and webs) can be modelled by allocating slab flexural stiffness to the main longitudinal members. Alternatively, a finer mesh can be superposed on the main frame structure.

4.2 Setting up the Model

The following procedure, also shown in Figure 2, can be used to set up any three dimensional frame model:

- Step 1. Set up the frames to model the transverse (out-of-plane) behaviour of the slab elements comprising the structure. In-plane flexural stiffness is rigid.
- Step 2. Place nodes at the intersection of the centroidal axes (y_c and z_c) of the section with the frame members.
- Step 3. Connect the frames longitudinally by adding members between the corresponding centroidal axis nodes on successive frames. These main longitudinal members represent the vertical and horizontal stiffnesses of the box section.
- Step 4. Edge beams and additional longitudinal members can be added if required, but must have zero area.

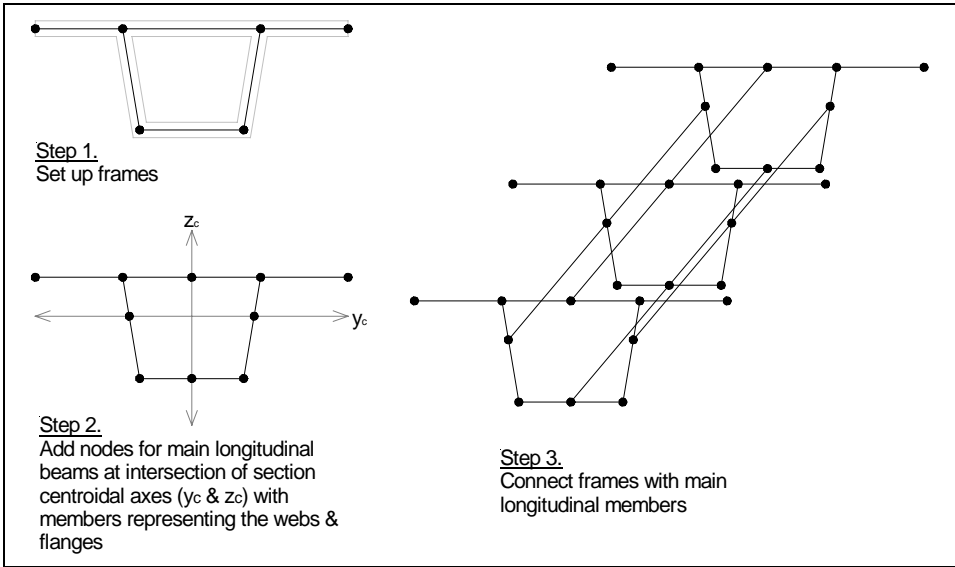


Figure 2: Setting up the model

4.3 Assigning Section Properties

Member section properties are assigned as follows:

Main longitudinal members modelling vertical stiffness (bending about y axis):

- Compression area (A) = 0
- Shear area (A_s) = area of web
- Bending stiffness about horizontal (y) axis (I_y) = I_y of effective beam (a proportion of the gross I_y of the section)
- Bending stiffness about vertical (z) axis (I_z) = stiffness of web = $bd^3/12$
- Torsion stiffness about longitudinal (x) axis (I_x) = assumed 0
(where b = height of web, d = web thickness)

Main longitudinal members modelling horizontal stiffness (bending about z axis):

- Compression area (A) = 0
- Shear area (A_s) = area of flange
- Bending stiffness about horizontal (y) axis (I_y) = stiffness of flange = $bd^3/12$
- Bending stiffness about vertical (z) axis (I_z) = I_z of effective beam (a proportion of the gross I_z stiffness of the section)
- Torsion stiffness about longitudinal (x) axis (I_x) = assumed 0
(where b = width of flange, d = flange thickness)

Slab members modelling frame action of box:

- Compression area (A) = area of slab = bd
- Shear area (A_s) = area of slab = bd (or assume shear rigid)
- Out-of-plane bending stiffness (I_y) = $bd^3/12$
- In-plane bending stiffness (I_z) = rigid
- Torsion stiffness (I_x) = torsion stiffness of continuous slab = $bd^3/6$
(where b = frame spacing, d = flange or web thickness)

4.4 Comments

The three dimensional frame modelling technique:

- quite accurately models the torsion/distortion behaviour of box structures;
- can be used to quickly model large and complex structures using relatively few members;
- directly outputs design bending moments and shear forces for longitudinal and transverse members.

Care must be taken to accurately model the support conditions, but the model readily allows diaphragms and bracing members to be added for correct support.

5. WORKED EXAMPLE – SINGLE CELL BOX GIRDER

Refer to the single cell box girder example shown in Figure 3. The box is simply supported, with supports directly beneath the webs.

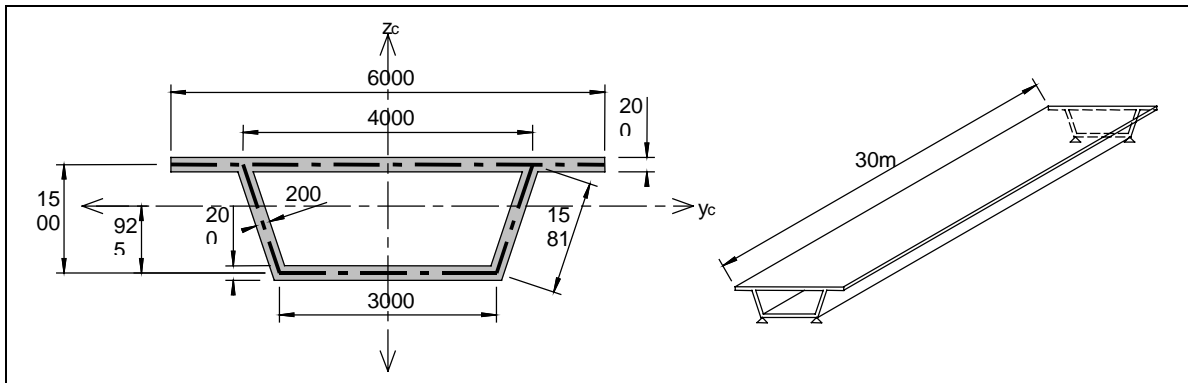


Figure 3: Single cell concrete box girder example

The gross section properties are:

- Area (A) = 2.390m^2
- Moment of inertia about the horizontal (y) axis (I_y) = 1.045m^4
- Moment of inertia about the vertical (z) axis (I_z) = 5.841m^4
- Modulus of elasticity (E) = 32000 MPa
- Poisson's ratio (ν) = 0.2
- Shear modulus (G) = 13333 MPa

The pure torsion stiffness of the section can be approximated using Equation (2), where:

$$A_e = \frac{1}{2}(4.000 + 3.000) \times 1.500 = 5.250\text{m}^2$$

Therefore:

$$J = C_t = \frac{4 \times 5.250^2}{\left(\frac{4000}{200} + \frac{3000}{200} + \frac{1581}{200} + \frac{1581}{200} \right)} = 2.170\text{m}^4$$

An eccentric point load is applied to the girder at midspan as shown in Figure 4. This load may be considered notionally equivalent to a concentric point load with an applied torque. If the point load (P) is 1000kN , and the eccentricity (e) is 2.0m , then the applied torque (T) is 2000kNm and the torsion diagram for the beam is as shown in Figure 4, where $T/2 = 1000\text{kNm}$, assuming the ends of the beam are fully torsionally restrained.

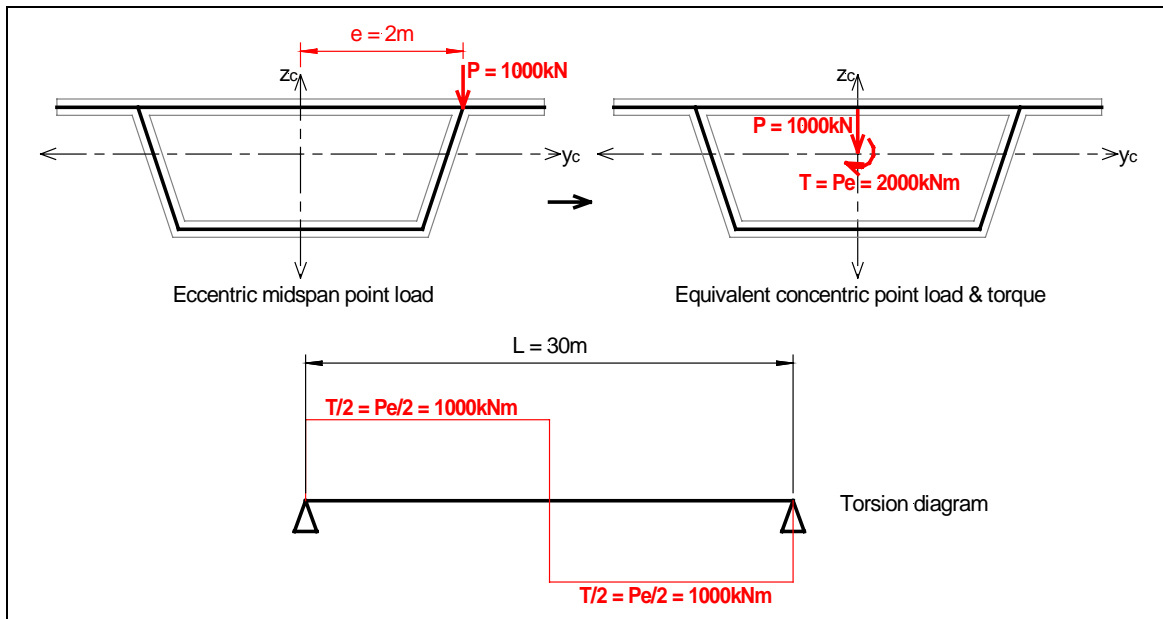


Figure 4: Eccentric midspan point load & corresponding torsion diagram

Using beam theory, the expected rotation at midspan may be calculated as such:

$$\theta = \frac{TL}{4GJ} = \frac{2000 \times 30}{4 \times 13333 \times 2.170} \times 10^{-3} = 0.000518 \text{ rad}$$

However, analysis of the box girder using finite elements reveals that the actual rotation is considerably larger.

5.1 Finite Element Analysis

The girder was modelled using thin shell finite elements with supports directly under each web to provide torsion restraint at the ends, as shown in Figure 5.

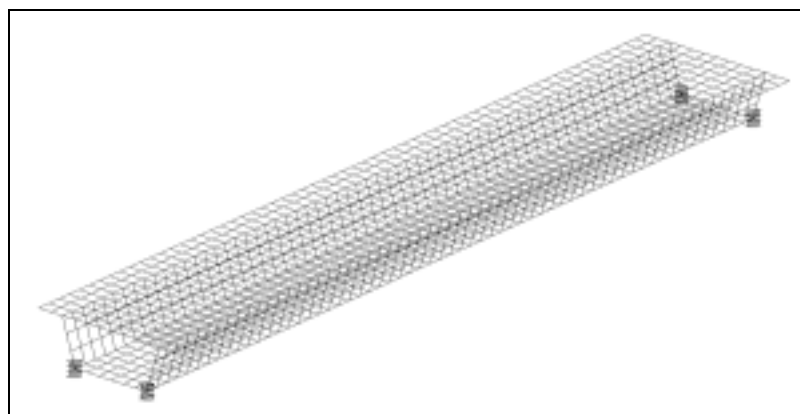


Figure 5: Three dimensional thin shell finite element model

The nodal displacements at midspan and at the end of the girder for the eccentric 1000kN point load are shown in Figure 6. It is evident that even though the dual supports provide torsional restraint at the ends of the girder, the flexibility of the webs allows the top flange to translate laterally relative to the bottom flange, hence the box deforms.

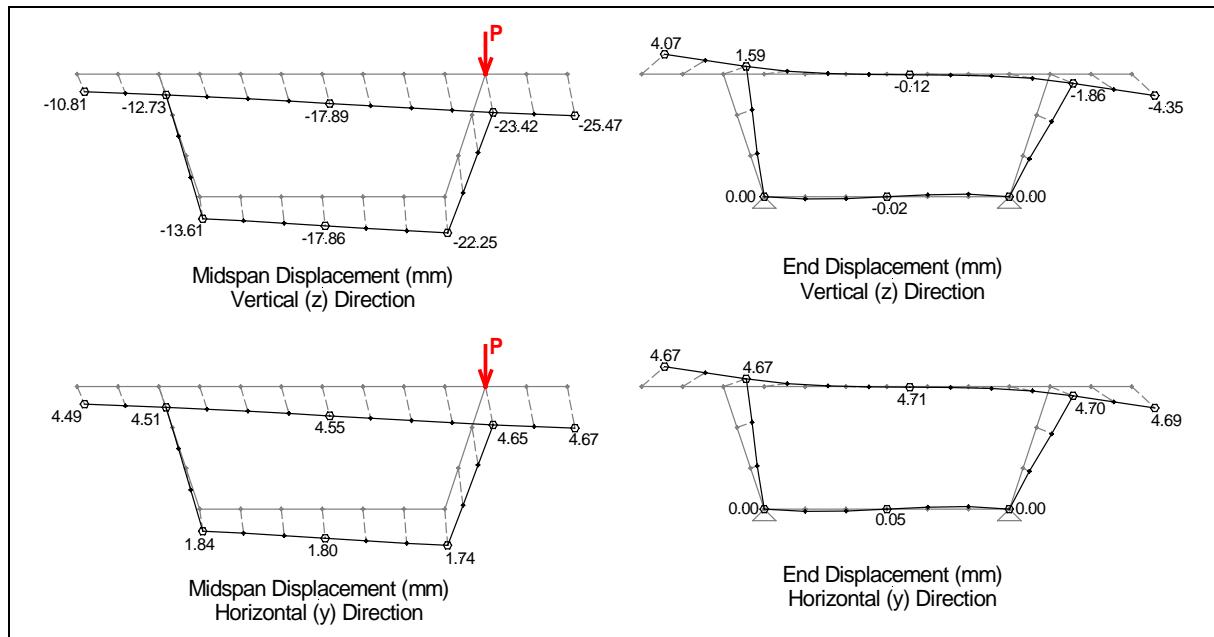


Figure 6: Displacements from FE Model

Although the deformation of the box means that there is not a uniform rotation of the section, the average rotation may be approximated by dividing the difference between the vertical displacements at the corners of the box by the horizontal distance between the corners. At midspan and for the top and bottom corners this gives the following rotations:

$$\text{Top corners} \quad \theta = \frac{\Delta_{z1,top} - \Delta_{z2,top}}{W_{top}} = \frac{23.42 - 12.73}{4000} = 0.00267 \text{ radians}$$

$$\text{Bottom corners} \quad \theta = \frac{\Delta_{z1,bottom} - \Delta_{z2,bottom}}{W_{bottom}} = \frac{22.25 - 13.61}{3000} = 0.00288 \text{ radians}$$

Hence the rotation at midspan from the finite element model is more than five times larger than that predicted by beam torsion theory (0.000518 radians).

5.2 Three Dimensional Frame Method

To model the box girder using the three dimensional frame method, two members representing the vertical stiffness and two members representing the horizontal stiffness were adopted. The longitudinal beams representing the vertical stiffness (Beams A and B, Figure 7) are located at the intersection of the horizontal centroidal axis (y_c) with the members representing the webs, and the longitudinal beams representing the horizontal stiffness (Beams C and D, Figure 7) are located at the intersection of the vertical centroidal (z_c) axis with the members representing the top and bottom flanges. These members have the following stiffness properties:

Beams A and B – left & right beams:

- Compression area (A) = 0
- Shear area (A_s) = area of web = $1.792 \times 0.2 = 0.358 \text{ m}^2$
- Bending stiffness about horizontal (y) axis (I_y) = $\frac{1}{2} \times 1.045 = 0.552 \text{ m}^4$

- Bending stiffness about vertical (z) axis (I_z) = $1/12 \times 1.5 \times 0.2^3 = 0.001$
- Torsion stiffness about longitudinal (x) axis (I_x) = assumed 0

Beam C – top beam (shaded area in Figure 7):

- Compression area (A) = 0
- Shear area (A_s) = area of top flange = $6.000 \times 0.2 = 1.200\text{m}^2$
- Bending stiffness about horizontal (y) axis (I_y) = $1/12 \times 6.0 \times 0.2^3 = 0.004$
- Bending stiffness about vertical (z) axis (I_z) = stiffness of shaded area (Figure 7) = 4.315m^4
- Torsion stiffness about longitudinal (x) axis (I_x) = assumed 0

Beam D – bottom beam (unshaded area in Figure 7):

- Compression area (A) = 0
- Shear area (A_s) = area of bottom flange = $3.000 \times 0.2 = 0.600\text{m}^2$
- Bending stiffness about horizontal (y) axis (I_y) = $1/12 \times 3.0 \times 0.2^3 = 0.002$
- Bending stiffness about vertical (z) axis (I_z) = stiffness of non-shaded area (Figure 7) = 1.527m^4
- Torsion stiffness about longitudinal (x) axis (I_x) = assumed 0

The main longitudinal beams are connected at regular intervals by frame members representing the slab. In this case, the 30m span was divided into 20 even segments at 1.5m intervals, giving 19 intermediate frames and two 0.75m wide end frames. The flanges and webs are all 200mm thick, therefore all the frame members have the same section properties as given below:

Intermediate frame members:

- Compression area (A) = area of slab = $1.5 \times 0.2 = 0.3\text{m}^2$
- Shear area (A_s) = rigid (large)
- Out-of-plane bending stiffness (I_y) = $1/12 \times 1.5 \times 0.2^3 = 0.001\text{m}^4$
- In-plane bending stiffness (I_z) = rigid (large)
- Torsion stiffness (I_x) = torsion stiffness of continuous slab = $1/6 \times 1.5 \times 0.2^3 = 0.002\text{m}^4$

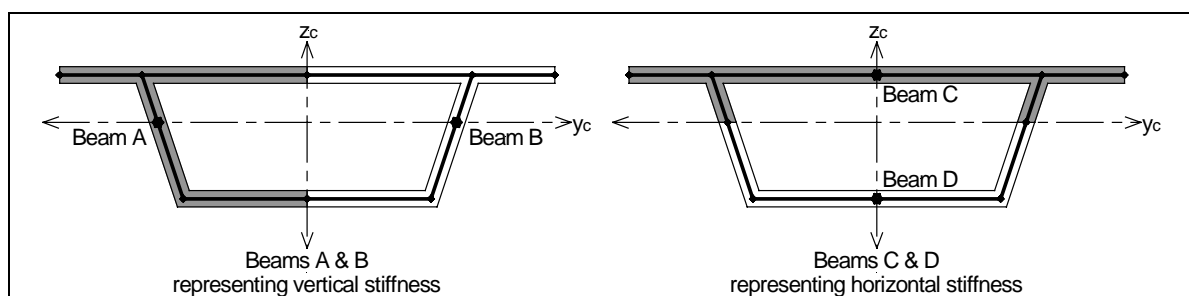


Figure 7: Main beams for three dimensional frame model

Figure 8 shows the fully assembled three dimensional frame model.

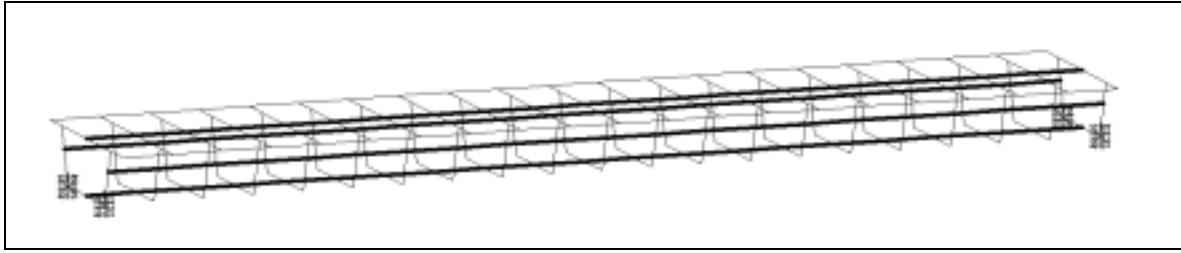


Figure 8: Three dimensional frame model

Figure 9 shows the displacements of the midspan and end frames. The values correlate well with those obtained from the finite element model and confirm the actual rotation is greater than that predicted by beam torsion theory (0.000518 radians). The following rotations at midspan are calculated for comparison with the finite element model using the previously described method:

$$\text{Top corners} \quad \theta = \frac{22.56 - 12.84}{4000} = 0.00243 \text{ radians}$$

$$\text{Bottom corners} \quad \theta = \frac{21.62 - 13.69}{3000} = 0.00264 \text{ radians}$$

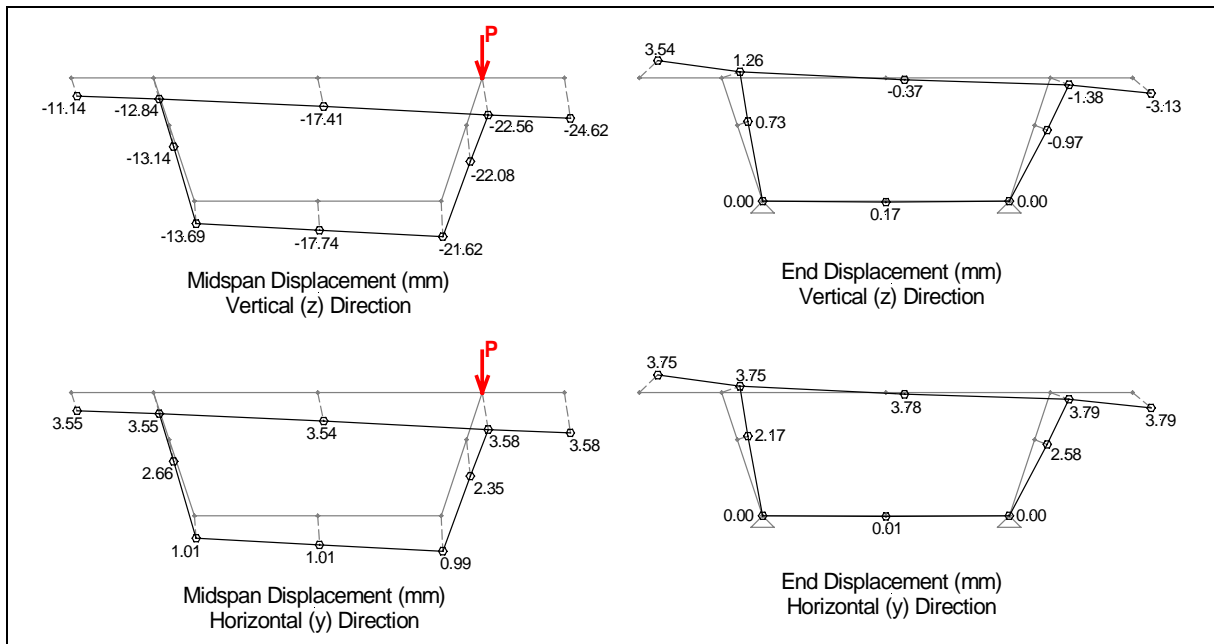


Figure 9: Displacements from three dimensional frame model

Bending moments and shear forces can be obtained directly from the model for each design beam. The maximum midspan bending moments about the horizontal axis for Beams A and B are 2843kNm and 4593kNm, therefore the distribution factors for the beams are 0.38 and 0.62 respectively. From these design moments and using Equation 3 below, it is possible to calculate the theoretical section stresses. Figure 10 compares the stresses obtained directly from the finite element model with those derived from the three dimensional frame model.

$$\sigma = \frac{My}{I} \quad (3)$$

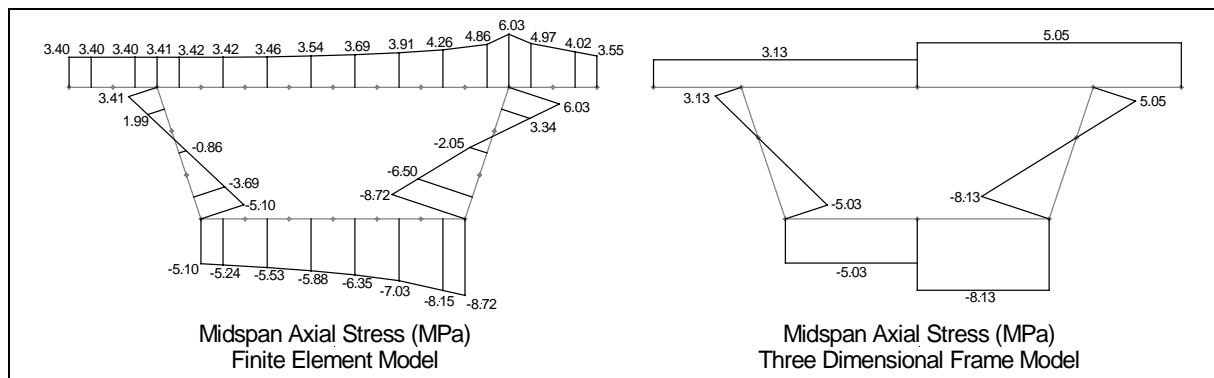


Figure 10: Comparison of axial stresses

The stresses from the top and bottom beams are not shown in Figure 10. These stresses are relatively small, but can be added to the stresses from the left and right beams for an even closer correlation of the stresses across the section.

5.3 Comparison of Methods

The main advantage of the three dimensional frame method over finite element analysis is its ability to accurately model the structure using relatively few members. In the example, the finite element model consisted of 1440 rectangular plate elements and 1464 nodes, while the three dimensional frame model consisted of just 330 beam members and 210 nodes.

The ACES software package, which was used for the finite element and three dimensional frame analyses, allows models of up to 6000 nodes, members, elements and loadings. Analysis of finite element models of this size or larger with moving vehicle load cases tends to be impractical or impossible with the existing ACES software, hence a more elaborate finite element package would be required. In comparison, the three dimensional frame model could easily be extended to include additional spans or complexity and still allow for moving vehicle load cases.

The other distinct advantage of the three dimensional frame model is the direct output of bending moments and shear forces. Stresses at any location and at any point in the section can be readily obtained from the calculated moments using beam stress theory.

5.4 Diaphragms

End diaphragms provide considerable restraint against rotation by preventing deformation of the box at the ends hence restricting lateral movement of the top flange. Figure 11 shows the results of adding end diaphragms to both the finite element and three dimensional frame models.

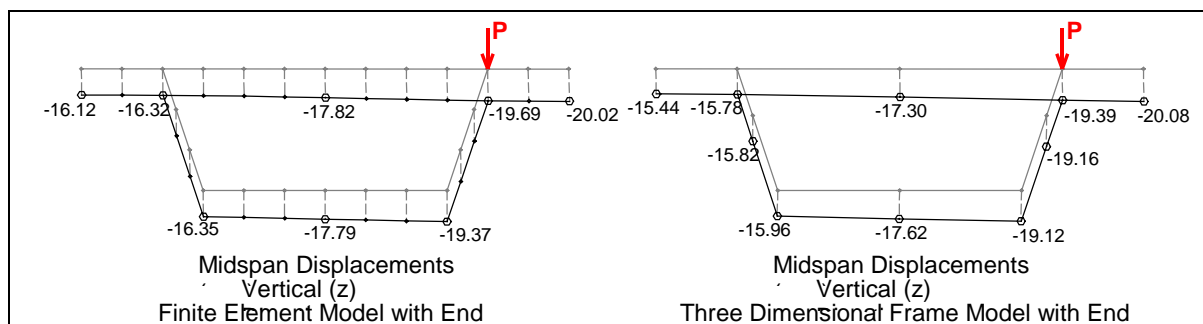


Figure 11: Midspan node displacements with end diaphragms

With end diaphragms, the calculated midspan rotations using the top corner nodes are 0.000842 and 0.000902 radians for the finite element and three dimensional frame models respectively. With end diaphragms the girder is approximately three times stiffer for rotation than without and is approaching the pure torsion stiffness of the box.

5.5 Summary

There is a good correlation between the finite element and three dimensional frame models, refer Table 1. Both models are capable of predicting the distortional behaviour of the box girder under torsional loading which is considerably different to the pure torsion case.

Modelling Method	Midspan Rotation (x 10 ⁻⁶ radians)	Ratio to Pure Torsion Case
Pure Torsion Case	518	1.0
FE Model – No diaphragms	2670	5.2
3D Frame Model – No diaphragms	2430	4.7
FE Model – With end diaphragms	842	1.6
3D Frame Model – With end diaphragms	902	1.7

Note: Rotations for the FE and 3D frame models are approximated using the displacements of the nodes at the top corners of the box girder.

Table 1: Summary of midspan rotations

6. SHEAR LAG STUDY

When using beam members to model the flexural stiffness of a section, the assumption is that plane sections will remain plane, however this is not necessarily true. The result of plane sections not remaining plane is a reduction in flexural stiffness, commonly known as shear lag. When modelling a structure using beam members, the effects of shear lag may be approximated by adopting an effective, or reduced, flexural stiffness for the members.

The effective stiffness of a member may be calculated by assuming reduced flange widths, however this requires some assessment to determine just how much of the flange should be considered effective. A more direct approach using finite element analysis is proposed, which also apportions stiffness between beams.

The method is based on a prescribed displacement, or forced yield, approach. The basic procedure for assessing the effective stiffness for each beam is:

- Step 1. Set up a finite element model of the structure. A single span should be sufficient.
- Step 2. Determine the location of the beams that are to be used to model the structure. For vertical flexure in a plane grillage or 3D model, it is likely that there will be a beam coincident with each web.
- Step 3. At a single cross section, place equal prescribed displacements to the structure at the location of each notional beam. For assessing bending about the horizontal (y) axis, the displacements will be in the vertical (z) direction, and for bending about the z axis, displacements will be in the y direction.
- Step 4. Analyse the finite element model to determine the loads required to generate the prescribed displacements.
- Step 5. Then find the gross effective stiffness as if the load was generated by a single line beam for the same prescribed displacement. Knowing the load (P), displacement (Δ), modulus of elasticity (E) and span length (L), back-calculate using beam theory to determine the effective stiffness of each beam. This will depend on how the model is supported, but for a simply supported model:

$$I_{effective} = \frac{PL^3}{48E\Delta} \quad (4)$$

This procedure may be applied in both the vertical and horizontal directions. The support conditions and loading pattern can be modified to assess a variety of situations.

6.1 Allocation of Shear Lagged Stiffness

The results of the above analysis can also be used to allocate stiffnesses between unsymmetrical beams, allowing for shear lag. For example if a single cell box has a gross transverse stiffness of 10 m^4 , for 100 mm lateral deflection it may require 65000 kN. The finite element model may give loads of 36000 kN and 24000 kN and for the top flange and lower flange respectively for simultaneous lateral deflections of 100 mm. This means that the gross effective lateral stiffness is $(36000 + 24000 = 60000) / 65000 = 9.2 \text{ m}^4$, and it will be allocated $(36000 / 60000) \times 9.2 = 5.5 \text{ m}^4$ to the top flange, and 3.7 m^4 to the lower flange.

7. APPLICATIONS

7.1 Mount Henry Bridge, Perth

Wyche Consulting Pty Ltd initially developed the three dimensional frame analysis method, for assessment of the Mount Henry Bridge in Perth as part of the Southwest Metropolitan Railway Project. The project brief required strengthening and widening of the existing box girder bridge for two new railway lines. A five span three dimensional frame model of the existing superstructure was set up (Figure 12), which was checked and calibrated against a single span finite element model. The frame model provided a tool for quick analysis of the structure, which allowed the client to investigate a number of construction options during the tendering period.

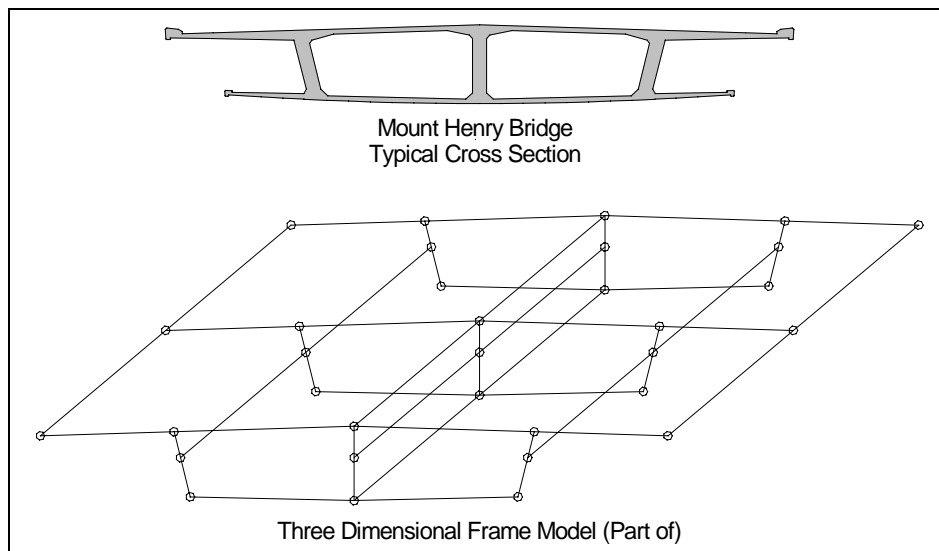


Figure 12: Modelling the Mount Henry Bridge, Perth

7.2 Composite Steel Box Girder – Design Verification

A design verification was carried out on two composite steel box girder bridges. The problem of modelling multiple cell steel boxes with a concrete deck was made even more difficult by the skewed ends, curved end diaphragms and internal cross bracing. The complex geometry of the bridges was able to be modelled quickly and accurately using the three dimensional frame method.

8. CONCLUSION

A new three dimensional frame analysis method, similar in principle to plane grillage analysis, has been described. The method can be used to quickly and accurately model the complex three dimensional distortional behaviour of box girders. Models require relatively few members, meaning that large structures can be modelled using small programs, such as ACES. The worked example given shows that the three dimensional frame method can be as accurate as a relatively fine mesh finite element model, but has the advantages of being quicker and easier to set up, and giving design actions directly. A simple method is described for assessing the effective stiffness of design beams allowing for shear lag, which can be used to refine the three dimensional model.

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