SYNOPSIS

Since it’s publication in 1976 up to the present day, Edmund Hambly’s book “Bridge Deck Behaviour” has remained a valuable resource for bridge engineers. During this period the processing power and storage capacity of computers has increased by a factor of over 1000 and analysis software has improved greatly in sophistication and ease of use. In spite of the increases in computing power, bridge deck analysis methods have not changed to the same extent, and grillage analysis remains the standard procedure for most structures.

In this paper the advantages and disadvantages of using more complex analysis procedures are examined. The following topics are covered:

- Alternatives to grillage analysis
- Comparison of design actions and deflections from alternative analysis methods
- Analysis of secondary effects
- Non-linear analysis
- Advanced analysis in the design office. Is it worth the effort?

Recommendations are presented that enable the advantages of advanced analysis techniques to be realised, whilst retaining the efficiency of grillage analysis.

1. INTRODUCTION

In the conclusion to the chapter on the finite element method in “Bridge Deck Behaviour”, Edmund Hambly wrote:

“The finite element method is the most powerful and versatile analytical method available at present because with a sufficiently large computer, the elastic behaviour of almost any structure can be analysed accurately. For this reason it is often requested by clients, or proposed to a client, to show that the most accurate analysis possible has been performed. Unfortunately, the method is cumbersome to use and is usually expensive. In addition, the choice of element type can be extremely critical and if incorrect, the results can be far more inaccurate than those predicted by simpler models such as grillage or space frame. However, perhaps the greatest drawback at present is that while the technique is developing so rapidly, the job of carrying out finite element computations is a full time occupation which cannot be carried out at the same time by the senior engineer responsible for the design. He is unlikely to have time to understand or verify the appropriateness of the element stiffnesses or to check the large quantity of computer data. This makes it difficult for him to place his confidence in the results, especially if the structure is too complicated for him to use simple physical reasoning to check orders of magnitude.” (1)
Since the publication of this book the processing power and storage capacity of desk top computers has increased by a factor of over 1000 and analysis software has improved greatly in sophistication and ease of use. In spite of the increases in computing power, bridge deck analysis methods have not changed to the same extent, and grillage analysis remains the standard procedure for most beam and slab structures.

In this paper the advantages and disadvantages of using more complex analysis procedures are examined. The following topics are covered:

- Alternatives to grillage analysis
  - 3D beam models
  - Plate models with downstand beams
  - Brick models
- Comparison of design actions and deflections from alternative analysis methods
- Analysis of secondary effects
  - Differential shrinkage and creep
  - Differential temperature
- Non-linear analysis
  - Effect of non-linear response on transverse distribution
  - Membrane action in deck slabs
- Advanced analysis in practice. Is it worth the effort?

2. ALTERNATIVES TO GRILLAGE ANALYSIS

2.1 Drawbacks of grillage analysis

Grillage analysis fails to deal with the following aspects of bridge deck behaviour:

- Transverse variation in the level of the neutral axis.
- Transverse and longitudinal in-plane forces
- Distortion of beam members
- Torsional and distortional warping effects
- Local bending effects
- The ends of skew decks cannot be modelled exactly

In addition the following effects are commonly neglected in the analysis of typical bridges:

- Non-linear response of beams and deck slabs
- Effects of construction sequence

2.2 3D beam models (downstand grillage)

A downstand grillage consists of beam elements located in space at the centroids of the members they represent, and connected with rigid links (Figure 1)
2.3 Plate models with downstand beams

This is similar to a downstand grillage, but with the transverse beam elements (representing the deck slab) replaced with plate elements.

2.4 Brick models

In a brick model all components of the deck are modelled with 3D brick elements (Figure 2), allowing all aspects of the deck geometry to be accurately modelled (with a sufficiently detailed mesh).

The aspects of bridge deck behaviour that can be modelled with these different types of analysis are summarised in Table 1:

<table>
<thead>
<tr>
<th>Feature</th>
<th>Downstand Grillage</th>
<th>Plates with Downstand beams</th>
<th>3D Brick models</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transverse variation in the level of the neutral axis.</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Transverse and longitudinal in-plane forces</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Distortion of beam members</td>
<td>✗</td>
<td>✗</td>
<td>✓</td>
</tr>
<tr>
<td>Torsional and distortional warping effects</td>
<td>✗</td>
<td>✗</td>
<td>✓</td>
</tr>
<tr>
<td>Local bending effects</td>
<td>✗</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Model skew decks exactly</td>
<td>✗</td>
<td>✗</td>
<td>✓</td>
</tr>
</tbody>
</table>

*Table 1: Features of bridge deck models*
3.0 COMPARATIVE ANALYSES

3.1 Bridge Deck Description

For the purposes of a comparison a simply supported single span beam and slab bridge deck was analysed under a uniformly distributed load and one lane of M1600 loading. The bridge consisted of 5 Type 4 Super T beams of 30 metres length, giving a deck width of 12.5 metres. The in-situ concrete deck was 160 mm thick with 800 x 250 mm parapet upstands, which were assumed to be composite with the deck. The beams were supported on single bearings, and were longitudinally fixed at one end and sliding at the other.

All the analyses were carried out using the general purpose finite element analysis package Strand7. A perspective view of the deck modelled with brick elements, and live load location, is shown in Figure 2.

3.2 Beam section properties

Four models consisting of single simply supported beams with associated transverse slab members, restrained against torsion, were analysed to verify that the section properties gave consistent results:

1) Plane grillage
2) Downstand grillage
3) Downstand beam with transverse beams replaced with plate elements
4) Beam and slab modelled with 20 node brick elements

The members in the grillage models (1 and 2) were modelled in accordance with Hambley’s recommendations (1):

- Longitudinal members were given the section properties of the composite precast and in-situ section.
• Transverse members were given a torsional stiffness equal to one half of the theoretical stiffness of a wide slab.
• The properties of the members between beam webs were based on the transverse properties of the composite beam and slab.

For the downstand grillage (2), rigid links were used to connect transverse and longitudinal members, which were placed at the level of their respective centroids.

For Model 3, using transverse plate elements, these elements contributed to the resistance to longitudinal bending. The longitudinal beam was therefore given the bending properties of the precast element only, and was connected to the plate elements with rigid links. The torsional stiffness of the longitudinal members was adjusted so that the total stiffness was equal to that of the composite box member in the actual structure. The properties used in the four models are shown in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Longitudinal beams:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>1.02E+00</td>
<td>1.02E+00</td>
<td>5.10E-01</td>
</tr>
<tr>
<td>IXX</td>
<td>3.48E-01</td>
<td>3.48E-01</td>
<td>1.09E-01</td>
</tr>
<tr>
<td>IYY</td>
<td>NA</td>
<td>3.47E-01</td>
<td>5.27E-02</td>
</tr>
<tr>
<td>J</td>
<td>1.74E-01</td>
<td>1.74E-01</td>
<td>1.74E-01</td>
</tr>
<tr>
<td>Shear Area</td>
<td>8.01E-01</td>
<td>8.01E-01</td>
<td>2.61E-01</td>
</tr>
<tr>
<td><strong>External Transverse Members</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>0.235</td>
<td>0.235</td>
<td>0.235</td>
</tr>
<tr>
<td>Width</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>A</td>
<td>1.76E-01</td>
<td>1.76E-01</td>
<td>1.76E-01</td>
</tr>
<tr>
<td>IXX</td>
<td>8.11E-04</td>
<td>8.11E-04</td>
<td>8.26E-03</td>
</tr>
<tr>
<td>IYY</td>
<td>NA</td>
<td>8.26E-03</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>5.24E-03</td>
<td>5.24E-03</td>
<td></td>
</tr>
<tr>
<td>Shear Area</td>
<td>1.47E-01</td>
<td>1.47E-01</td>
<td></td>
</tr>
<tr>
<td><strong>Internal Transverse Members</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth</td>
<td>0.16</td>
<td>0.16</td>
<td>0.16</td>
</tr>
<tr>
<td>Width</td>
<td>0.75</td>
<td>0.75</td>
<td>0.75</td>
</tr>
<tr>
<td>A</td>
<td>0.12</td>
<td>0.12</td>
<td></td>
</tr>
<tr>
<td>IXX</td>
<td>0.16</td>
<td>0.16</td>
<td></td>
</tr>
<tr>
<td>IYY</td>
<td>NA</td>
<td>5.63E-03</td>
<td></td>
</tr>
<tr>
<td>J</td>
<td>0.32</td>
<td>0.32</td>
<td></td>
</tr>
<tr>
<td>Shear Area</td>
<td>2.29E-03</td>
<td>2.29E-03</td>
<td></td>
</tr>
</tbody>
</table>

*Table 2: Beam section properties*

Section properties for Model 4 brick elements and Model 3 plate elements are determined by the member dimensions.

The following loads were applied to check the behaviour of the beam elements under vertical loads, torsional loads, and combined vertical and torsion loads:

1) Uniform distributed load (applied as member self weights)
2) Uniform torsional load, applied as equal and opposite vertical loads applied to the ends of the transverse members
3) Uniform torsional plus vertical load, applied as vertical loads applied to one end of the transverse members.

The bending stiffness of the beams was set to give close agreement for deflection under uniform load. Deflections along one edge of the top slab for the torsional load cases are shown in Figure 3, and deflections across the top slab at mid-span are shown in Figure 4. It can be seen that the results for combined torsion and vertical load were in good agreement for all models. Under pure torsion load the grillage models gave a greater edge deflection at mid-span than the brick model, but the difference in deflection between support and mid span was similar. The difference in behaviour is because the transverse members in the grillage analysis do not directly interact. The plate slab model was significantly less stiff torsionally than the other models, because the high transverse bending stiffness of the longitudinal beams was not modelled in this analysis.

![Figure 3: Beam deflections under torsional loads](image)

![Figure 4: Beam mid-span deflections under torsional loads](image)

3.3 Deck Models

One lane of M1600 loading was applied to an outer lane, using the following models:

1a: Plane grillage, wheel loads distributed to longitudinal members
1b: Plane grillage, wheel loads distributed to nodes at the ends of transverse members
1c: Plane grillage, wheel loads applied to transverse members
2: Downstand grillage
3: Downstand beam with plate elements
4: Brick elements

In models 2 to 4 the wheel loads were distributed to the nearest nodes, as for model 1b. Deflections and stresses at mid span are plotted across the width of the deck in Figures 5 to 8.

3.4 Deflections

Deck deflections at mid-span under M1600 loading are shown in Figure 5. It can be seen that:

- Application of the wheel loads to the longitudinal grillage members (Grillage 1a) gave a significantly different deflected shape to the other grillage models.
- Distribution of the wheel loads to transverse beam nodes (Grillage 1b) gave almost identical deflections to application of the loads to the transverse members (Grillage 1c)
- The downstand grillage (Grillage 2) and the plate slab models gave a greater distribution of the load, with reduced deflection under the load and slightly greater deflections at the unloaded beams.
- The brick model gave the greatest distribution of the load, with maximum deflections being reduced by about 15%

![Figure 5: Mid-span deflections of decks](image-url)
Figure 6: Maximum Bending Moments

Figure 7: Maximum Top Face Stress

Figure 8: Maximum Bottom Face Stress
3.5 Bending Moments and Stresses

Maximum bending moments, top face stresses, and bottom face stresses are shown in Figures 6, 7 and 8 respectively. The results follow a similar trend to the mid-span deflections:

- The three plane grillage runs all gave similar results for maximum stress and bending moment.
- The downstand grillage and plate slab models gave slightly lower bending moments and stresses.
- The brick model gave lower bending moments for internal beams, with the external beams attracting higher bending moments.
- Because of the composite action of the parapet upstand, and the resulting higher neutral axis, the brick model attracted higher bending moments to the stiff edge beams, but gave significantly lower stresses in all the loaded beams, with the maximum stress in the bottom face being reduced by about 15%.

3.6 Transverse slab bending moments

Maximum transverse moments in the top slab are shown in Table 3. In the grillage analyses the maximum sagging moments were found to increase only slightly when the wheel loads were applied to the transverse members, indicating that the global effects dominated the behaviour. The grillage models and the plate model gave very similar results, with the brick model giving slightly lower moments.

For the hogging moments all the grillage models gave substantially higher moments than the plate or brick models. The reasons for the high hogging moments in the grillage models were:

- The grillage models did not model the support from the twin webs of the longitudinal beams, but rather modelled the beams as line supports, with short stiff transverse members.
- The plate and brick models include the longitudinal distribution of wheel loads through the slab, whereas the grillage models transfer the wheel loads directly to longitudinal beams.

<table>
<thead>
<tr>
<th>Model</th>
<th>Maximum Transverse Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Positive</td>
</tr>
<tr>
<td>Grillage 1a</td>
<td>25.6</td>
</tr>
<tr>
<td>Grillage 1b</td>
<td>29.1</td>
</tr>
<tr>
<td>Grillage 1c</td>
<td>29.6</td>
</tr>
<tr>
<td>Grillage 2</td>
<td>30.0</td>
</tr>
<tr>
<td>Plate Slab</td>
<td>30.8</td>
</tr>
<tr>
<td>Brick</td>
<td>27.5</td>
</tr>
</tbody>
</table>

Table 3: Maximum transverse moments
3.7 Torsional Stiffness and Cracking

The analyses presented in this paper used the torsional stiffness of the uncracked concrete section, since the maximum torsional moments were well below the cracking torque of the pre-stressed beams. In practice some bridge decks will have much higher torsional moments, and the torsional stiffness of beams may reduce to as little as one-tenth of the uncracked value, Warner et al(4). It is therefore important that the effect of torsional stiffness on load distribution should be considered, and that the torsional stiffness of members should be reduced in accordance with the Australian Bridge Design Code (3) requirements where appropriate.

3.8 Data preparation, execution time and post-processing

Grillage models may be prepared automatically for standard bridge types very quickly. Preparation of brick models will take longer, but with the use of standard elements it will not require a significant increase in the overall design process.

Analysis time for the linear analysis with two load cases considered in this study was 1 or 2 seconds for the grillage models, about 10 seconds for the plate model, and about 2 minutes for the brick model. Use of brick elements is therefore perfectly feasible as far as computing time is concerned, particularly for final design purposes, where the total number of load cases may be reduced with preliminary analysis using simpler models. Non-linear analysis of the brick model would require substantially greater computing time, but this would be quite feasible, particularly for checking specific load cases.

Member actions are output directly from the analyses using beam and plate elements. Extraction of member actions from brick element results requires considerably greater effort, nonetheless with the use of appropriate software the process may be automated, so that it may be carried out without a large increase in engineering time.

4.0 ANALYSIS OF SECONDARY EFFECTS

Three dimensional computer models (i.e. the downstand beam and brick models) allow secondary effects to be modelled directly in the bridge deck model, allowing these effects to be combined and enveloped with the primary load effects. This provides a considerable saving in time and effort, and reduces the risk of error. Effects that may be treated in this way include:

- Longitudinal differential shrinkage; a longitudinal strain may be applied to the top slab members.
- Transverse differential shrinkage; effects due to wide decks being cast in stages, or due to parapet or cantilever shrinkage, may be easily modelled.
- Differential creep; the variation in elastic modulus may be modelled over time, allowing creep effects to be accurately analysed, and the effect of prestress losses automatically included.
- Differential temperature; differential temperatures may be applied directly to plate and brick members, allowing the distribution of temperature effects through the deck to be easily analysed.
Secondary prestress moments; in continuous structures secondary prestress moments will be automatically included in the analysis, as will secondary moments due to differential strain effects and prestress losses.

5.0 NON-LINEAR ANALYSIS

5.1 Transverse Distribution

The transverse distribution of vehicle loads may be significantly affected at the ultimate limit state by the reduction in stiffness of cracked reinforced concrete members. This aspect of behaviour is most easily investigated using a grillage model, where the moment-curvature behaviour of the beam members may be specified, allowing a non-linear analysis to be carried out easily and quickly. Plate members also allow non-linear bending behaviour to be captured comparatively easily, but the complexity of models using brick elements would need to be increased considerably to model the non-linear behaviour adequately.

5.2 Membrane Action

Membrane arching action is known to significantly reduce bending moments and reinforcement stresses in bridge decks, but this effect is usually ignored, due to the difficulty in modelling the behaviour reliably. It is necessary to model three dimensional effects, and also the different behaviour of reinforced concrete in compression and tension. It is feasible to carry out this analysis using non-linear brick elements to model the deck slab, but as noted above, a substantial increase in the complexity of the model is required to model the non-linear bending behaviour adequately.

6.0 ADVANCED ANALYSIS IN PRACTICE

The potential advantages of replacing grillage analysis with finite element plate or brick models may be summarised as follows:

- Better modelling of the transverse distribution of live loads may significantly reduce maximum design stresses in longitudinal members.
- Distribution of wheel loads is more accurately modelled and combined with global effects, resulting in a more accurate estimate top slab bending moments, without the need for introducing separate local analyses.
- Secondary effects such as differential temperature and shrinkage may be combined and enveloped with live load effects using the same model.
- Features such as the ends of skew decks and link slabs between simply supported decks may be modelled more exactly, including three dimensional effects.

The disadvantages of the use of the finite element method for bridge deck analysis listed by Hambly have largely been removed with the availability of powerful desk top computers and sophisticated finite element packages:

- Finite element models may now be produced and analysed using standard computing equipment in a shorter time than a grillage analysis would have taken in the recent past.
• The accuracy of complex models may be checked against grillage analysis, or individual elements may be checked against simple analysis methods.
• Three dimensional contour plots of stresses or plots of the deformed shape of structures are easily produced, allowing engineers not directly involved in the analysis to review the results, and check the validity of the model.

Remaining drawbacks to the use of finite element analysis, particularly using brick elements, are:

• It is more difficult to extract member actions, particularly for large elements such as bridge beams.
• The design engineer must be trained in the use of complex software to use it efficiently.
• The verification process may be more difficult, particularly if detailed analysis has resulted in lower design actions than a simpler analysis.

In order to take advantage of the benefits of advanced analysis techniques, without unnecessary complication of the design process, the following recommendations are made:

• Use of a plate slab model with longitudinal beam members is recommended as a standard analysis procedure.
• Pre and post-processor software, specifically designed for bridge decks, can greatly reduce analysis and design time, and allow the efficient use of general purpose analysis software.
• Brick models may be used to further refine the design, or to investigate the behaviour of non-standard features.
• Non-linear analysis and consideration of slab membrane action provides the potential for significant refinement of deck slab design.

REFERENCES

1 HAMBLY, E.C. Bridge Deck Behaviour, CHAPMAN & HALL, 1976
2 Australian Bridge Design code, AUSTROADS, 1992
4 WARNER, R.F. Concrete Structures, LONGMAN, 1998